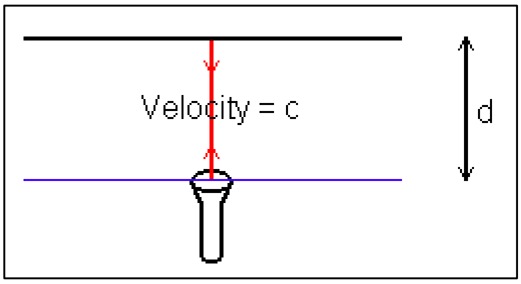
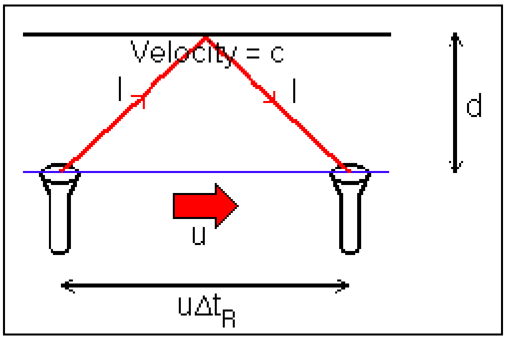
Worksheet 1

From his vantage point on the train, Mr. Flat Hat sees this. A simple calculation gives the round trip time Δt_F (subscript F for Flat Hat!) (the time it takes for the light to go from the bottom mirror to the top one and back again).



Equation 1

Round-trip time : Δt_F = _______



Meanwhile, over on the platform, Mr. Round Hat observes something a bit different. By the time the light has bounced back to the source, the source has travelled a distance $u\Delta t_R$, so the light has travelled further. Mr. Round Hat still observes the speed of light to be c, so perceives a longer round-trip time.

Equation 2

Use Pythagoras' theorem to find the distance l in terms of d, u and delta_t_R.

l = ________________________

Equation 3

Now, find the round-trip time as seen by Mr. Round Hat.

$$\Delta t_R = \frac{2l}{c} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Worksheet 1

Equation 4

Substitute Equation 1 into Equation 3 to get rid of d

$$\Delta t_R = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Equation 5 : This is it!!!

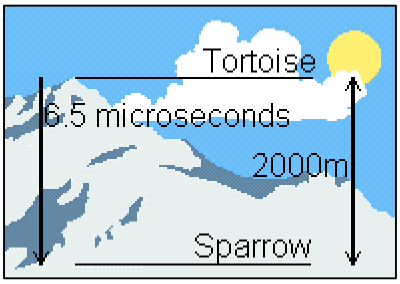
Square both sides of Equation 4 and solve for Δt_R

$$\Delta t_R = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Notice that Δt_R is given by a number times Δt_F .
This number is called the ***gamma factor***, as you will see elsewhere.

Worksheet 2

The mountain is 2000m high, and the muons take 6.5 microseconds (ie 6.5×10^{-6} seconds) to travel from the top to the bottom. They arrive at the top of the mountain at a rate of 563 per hour.
The muons have a ***halflife*** ($t_{\frac{1}{2}}$) of 1.5 microseconds, equivalent to a ***lifetime*** ($t_{\frac{1}{e}}$) of 2.16 microseconds.



The difference between the halflife and the lifetime is that after one halflife, ½ of the muons will have decayed, whereas after one lifetime, $^1/_e$ of the muons will remain. It is easier to work in lifetimes, as calculators are fitted with *e* and *ln* buttons!

Part 1

Think before you stick the numbers in!!

- Should the rate of arrival at the bottom of the mountain be larger or smaller than the rate at the top?
- How much larger or smaller?
- How many muons would be left if the muons took 2.16 microseconds to make the journey? What if they took 4.32 microseconds?

'Expected' rate at bottom of mountain = _____ x e ________

Parts 2 & 3

The tortoise actually observes 400 muons per hour at the bottom of the mountain.

- Hint : Remember that you have a *ln* button on your calculator!
- Calculate the 'actual' lifetime in the muons' frame of reference, then compare this with the 'expected' lifetime to find the gamma factor.
- Will the muons perceive the mountain to be taller or shorter?

Rate = _____ = e __________

therefore

ln _____ = ____/____

and

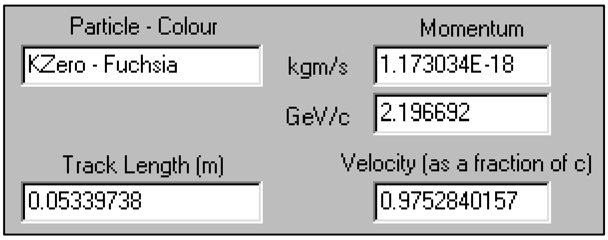
gamma = _______

so

Height in muons' reference frame = __________

Worksheet 3

By clicking on the K^0, you should have found out some information about it. As it is travelling so rapidly (note its velocity!) it is subject to relativistic effects such as time dilation (as were the muons coming down the mountain).



Part 1

In order to calculate the K^0's lifetime in its own inertial frame, we first need to find its gamma factor, and its lifetime in our inertial frame.

Insert the ***velocity*** of the K^0 in the equation you solved on Worksheet 1 to find its ***gamma factor***.

g = ____________________________

Part 2

Now, use the ***velocity*** and the ***length*** of the K^0's track to find its ***lifetime in our inertial frame***.

- Hint : Don’t be scared – you probably first came across this equation in Primary Six!!

Lab lifetime = ______________

Part 3

Finally, use the gamma factor to find its lifetime in its own inertial frame (i.e. its lifetime when it is at rest.

- Hint : Above all, ***think about it***!! Will the K^0’s rest lifetime be longer or shorter than the measured lifetime in the lab?

Rest lifetime = ________________________